105台大資演

1.
(a) D (b) G (c) C (D) B (e) C (f) H

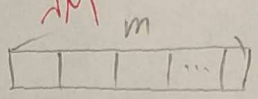
2. (a) $O(n^{\log_8 3}) + O(n^{\frac{1}{3}}) = \underline{\Theta(n^{\log_8 3})}$ //

(b) $T(n) = T(n^{\frac{1}{2}}) + 1$, 令 $n = 2^{2^k}$, $T(2^{2^k}) = T(2^{2^{k-1}}) + 1$

令 $T(2^{2^k}) = a_k$, $a_k = a_{k-1} + 1$, $a_k = k$ $T(n) = \underline{\Theta(\lg\lg n)}$ //

(c) $O(n^{\log_2 8}) + \Theta(1) = \underline{\Theta(n^3)}$ // $\frac{n^3}{m}$

3. m個element、n個slot



(a) hash找index => O(1)

在slot內找(slot為AVL tree) => 每個slot內平均有 $\frac{n}{m}$ 個element

AVL tree search time : $O(\lg(\frac{n}{m}))$

(b) $m = n^2$ ① 共有 $\binom{n}{2}$ pair 可能碰撞，每對機率為 $\frac{1}{m}$

② $\frac{n \times (n-1)}{2} \times \frac{1}{m}$ 趨近0(1)，則 m 至少要為 n^2

↳ 兩兩碰撞處理可在O(1)解決

(c)

0	1	2	3	4	5	6
38	8	16		22		11

① 22/5 = 3…1 collison

H(22 mod 5) = 3

1+3 = 4

② 11/5 = 1…4 collison

H(11 mod 5) = 2

4+2 = 6.

4

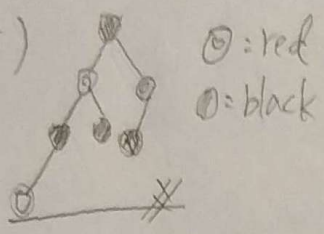
(a) ① 執行 inorder algorithm，output一個陣列

② 檢查陣列元素是否小→大排序 {是 return true / 不是 return false

4.

(b)
```
catalan_number(int n)
    if(n==0||n==1) return 1;
    int sum=0;
    for(i=0;i<n;i++)
     sum += catalan(i)+catalan(n-i)
    return sum;
```

(c)

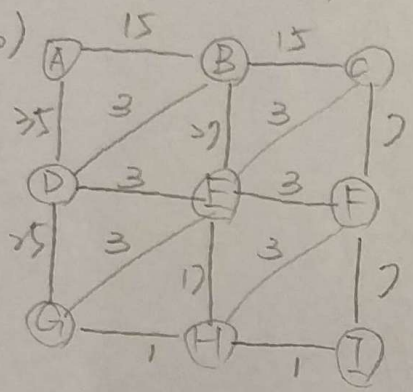


⊙: red
●: black

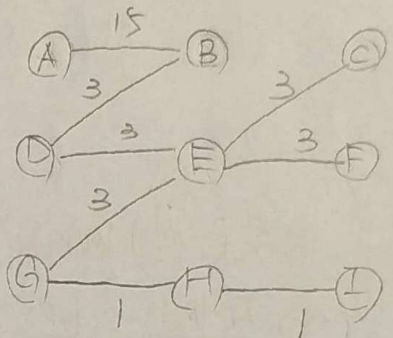
5.

(a) A→D→E→G→H ⇒ 27 # greedy找

(b)



用kruskal's algorithm



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6.

$\Phi(H)=\{D_0, D_1, \cdots, D_n\}$

D_i: 執行完operation的heap

C_i: 執行第i個instruction之實際成本，worst case皆$O(\log n) \Rightarrow C_i \le k\log n$

定義 $\overline{C_i}$ 為 amortized cost，$\overline{C_i} = C_i + \Phi(D_i) - \Phi(D_{i-1})$

$\Phi(D_i)$ 皆會 ≥ 0 (成本)

$\overline{C_i} = C_i + \Phi(D_i) - \Phi(D_{i-1}) = k\log n + k\log n = O(\log n)$ (insert)

$\overline{C_i} = C_i + \Phi(D_i) - \Phi(D_{i-1}) = k\log n - k\log n = O(1)$ (extract-min)

2.

1a)

(1) 0 (2) 1 (3) $L(i+1,j-1)+2$ (4) $\max[L(i,j-1), L(i+1,j)]$

1b)

str1 / str2	C	A	B	D	A	A	C	B	A	D	F	A
A	↑0	↖1	← 1	← 1	↖1	↖1	← 1	← 1	↖1	← 1	← 1	↖1
F	↑0	↑1	↑1	↑1	↑1	↑1	↑1	↑1	↑1	↑1	↖2	← 2
D	↑0	↑1	↑1	↖2	← 2	← 2	← 2	← 2	← 2	↖2	← 2	← 2
A	↑0	↖1	↑1	↑2	↖3	↖3	← 3	← 3	↖3	← 3	← 3	↖3
B	↑0	↑1	↖2	← 2	↑3	↑3	↑3	↖4	← 4	← 4	← 4	← 4
C	↖1	↑1	↑2	↑2	↑3	↑3	↖5	↑4	↑4	↑4	↑4	↑4
A	↑1	↖2	↑2	↑2	↖3	↖4	↑4	↑4	↖5	← 5	← 5	↖5
A	↑1	↖2	↑2	↑2	↖3	↖4	↑4	↑4	↖5	↑5	↑5	↖6
D	↑1	↑2	↑2	↖3	↑3	↑4	↑4	↑4	↑5	↖6	← 6	↑6
B	↑1	↑2	↖3	↑3	↑3	↑4	↑4	↖5	↑5	↑6	↑6	↑6
A	↑1	↖2	↑3	↑3	↖4	↖4	↑4	↑5	↖6	↑6	↑6	↖7
C	↖1	↑2	↑3	↑3	↑4	↑4	↖5	↑5	↑6	↑6	↑6	↑7

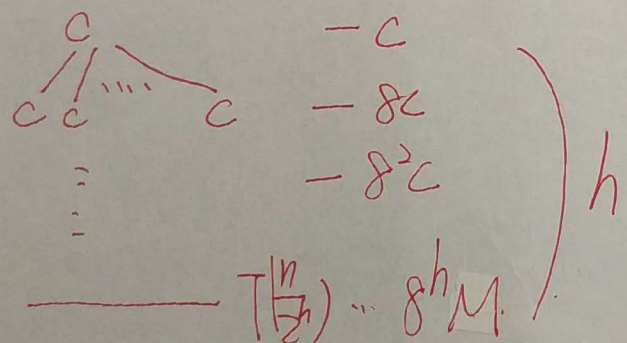
① 令 str2 = str1 之倒序.

② 做 LCS

ADACADA //

2.

1c)



$T(\frac{n}{2^0}) \cdots T(\frac{n}{2^h}) \cdots 8^h M$

每次要 $\frac{n}{2}$，共要 $\frac{n}{2^h}$ 次 ⇒ 又條件為 $(\frac{n}{2^h})^2 \le M$

則 $h = \lg \frac{n}{\sqrt{M}}$

$T(n) = \Theta(8^h M) = \Theta(8^{\lg \frac{n}{\sqrt{M}}} \cdot M) = \Theta((\frac{n}{\sqrt{M}})^3 \cdot M)$

$= \Theta(\frac{n^3}{\sqrt{M}})$.