1.

$$\left[\begin{array}{ccc|ccc} r^2-2 & -r & 1 & 1 & 0 & 0 \\ -r & 1 & 0 & 0 & 1 & 0 \\ 3-r^2 & r & -1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} r^2-2 & -r & 1 & 1 & 0 & 0 \\ -r & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 0 & -r & 1 & -r^2+3 & 0 & -r^2+2 \\ 0 & 1 & 0 & r & 1 & r \\ 1 & 0 & 0 & 1 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 3 & r & 2 \\ 0 & 1 & 0 & r & 1 & r \\ 1 & 0 & 0 & 1 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & r & 1 & r \\ 0 & 0 & 1 & 3 & r & 2 \end{array}\right] \#$$

2. ① $n\times n$，② 可對角化，③ 複數矩陣，④ eigenvalue $=0$ & 1 $\Rightarrow \det(A)=0 \Rightarrow$ singular

①、② $\Rightarrow A = PDP^{-1}$

$\Rightarrow A^k = PD^kP^{-1}$　　$k=2$ #

極小多項式 $min_A(x) = (x-0)(x-1)$

$\Rightarrow A(A-I)=0$

$A^2-A=0$　則 $A^2=A$　$k=2$ #

3. $L: x-y=0$　$\begin{bmatrix} x \\ y \end{bmatrix} = A$

L的投影矩陣 $= \begin{bmatrix} x \\ y \end{bmatrix}\left(\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}\right)^{-1}\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} \frac{x^2}{x^2+y^2} & \frac{xy}{x^2+y^2} \\ \frac{xy}{x^2+y^2} & \frac{y^2}{x^2+y^2} \end{bmatrix}$

4. 令 R^2 之標準 basis 為 $B=\{(1,0),(0,1)\}$

轉換後之 basis 為 $\gamma = \{(1,0),(v_1,v_2)\}$ 要找這個

原本 $x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $y=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ on B，坐標轉換矩陣 $[I]_\gamma^B = \begin{bmatrix} 1 & v_1 \\ 0 & v_2 \end{bmatrix}$

轉後 $x=\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$, $y=\begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$ on γ

$[x]_B^B = [I]_\gamma^B [x]_\gamma^\gamma \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & v_1 \\ 0 & v_2 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$ $\begin{cases} x_1 = s_1+s_2v_1 \\ x_2 = s_2v_2 \end{cases}$

$[y]_B^B = [I]_\gamma^B [y]_\gamma^\gamma \Rightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & v_1 \\ 0 & v_2 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$ $\begin{cases} y_1 = t_1+t_2v_1 \\ y_2 = t_2v_2 \end{cases}$

s,t 是實數

$x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2 = s_1t_1 + s_2t_2$

$\Rightarrow \underline{(v_1-v_2)}(s_1t_2) + \underline{(v_1-v_2)}s_2t_1 + \underline{(v_1^2-2v_1v_2+4v_2^2)}\cdot s_2t_2 = s_2t_2$

取 $v_1=v_2$

則這項 $=1-v_1^2-2v_1^2+4v_1^2 \Rightarrow v=\pm\frac{1}{\sqrt{3}}$ # $\Rightarrow \vec{v} = \pm\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$

6. 總數: 2^{m^2}

具反身/非反身: $2\times 2^{m^2-m}$] $\underline{2^{m^2}-2^{m^2-m+1}}$ ✓

7.

$(p\vee q)\wedge(\neg p\vee r)$

$=(p\wedge\neg p)\vee(q\wedge\neg p)\vee(p\wedge r)\vee(q\wedge r)$

$=[q\wedge(\neg p\vee r)]\vee(p\wedge r)$ ✓ 答案不唯一

8.

$a_{n+2}-a_{n+1}-a_n=0$

$\alpha=\dfrac{1\pm\sqrt{5}}{2} \Rightarrow a_n=C_1\cdot\left(\dfrac{1+\sqrt{5}}{2}\right)^n+C_2\cdot\left(\dfrac{1-\sqrt{5}}{2}\right)^n$

$a_0=C_1+C_2$

$a_1=\left(\dfrac{1+\sqrt{5}}{2}\right)C_1+\left(\dfrac{1-\sqrt{5}}{2}\right)C_2$

$C_2=\dfrac{1+\sqrt{5}}{2\sqrt{5}}a_0-\dfrac{1}{\sqrt{5}}a_1=\dfrac{(1+\sqrt{5})a_0-2a_1}{2\sqrt{5}}$

$C_1=\dfrac{2\sqrt{5}a_0}{2\sqrt{5}}-\dfrac{(1+\sqrt{5})a_0-2a_1}{2\sqrt{5}}=\dfrac{-1+\sqrt{5}+2a_1}{2\sqrt{5}}$

9.

n-1若和n不互質 ⇒ 則 $\exists x\neq 1 \ni (n-1|x)\wedge(n|x)$

① 則 $[n-(n-1)]|x$

② 得 $1|x$ ⇒ 沒有可滿足 ~~x~~ $\exists x\neq 1 \ni \gcd(n-1,n)=x$

10. bipartite(双分圖)或 2-color.

11.

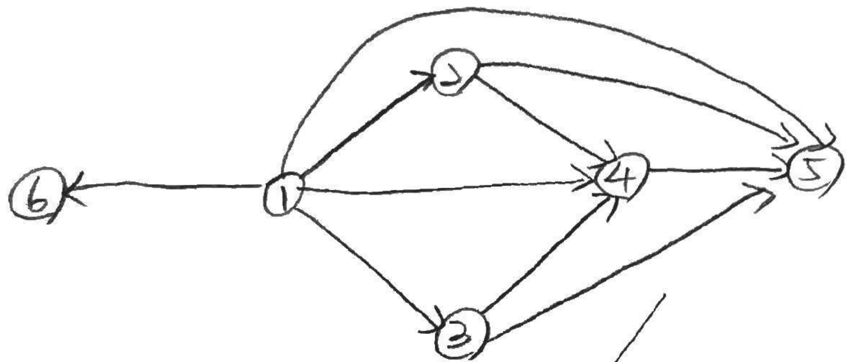
① $\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{\frac{n-1}{2}}+\underbrace{\binom{n}{\frac{n+1}{2}}+\binom{n}{\frac{n+3}{2}}+\binom{n}{\frac{n+5}{2}}+\cdots+\binom{n}{\frac{n+n}{2}}}_{\text{考慮加入後半}}$

= 則整個為 $\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{n}=2^n$

② 又因 $\binom{n}{0}=\binom{n}{n}$, $\binom{n}{\frac{n-1}{2}}=\binom{n}{\frac{n+1}{2}}$, ...

剛好為一半 $\Rightarrow 2^{n-1}$ #

12.



考慮① ⇒ 加入 $\overline{15}$

考慮② ⇒ 加入 $\overline{25}$

考慮③ ⇒ 加入 $\overline{35}$