Network System Capstone
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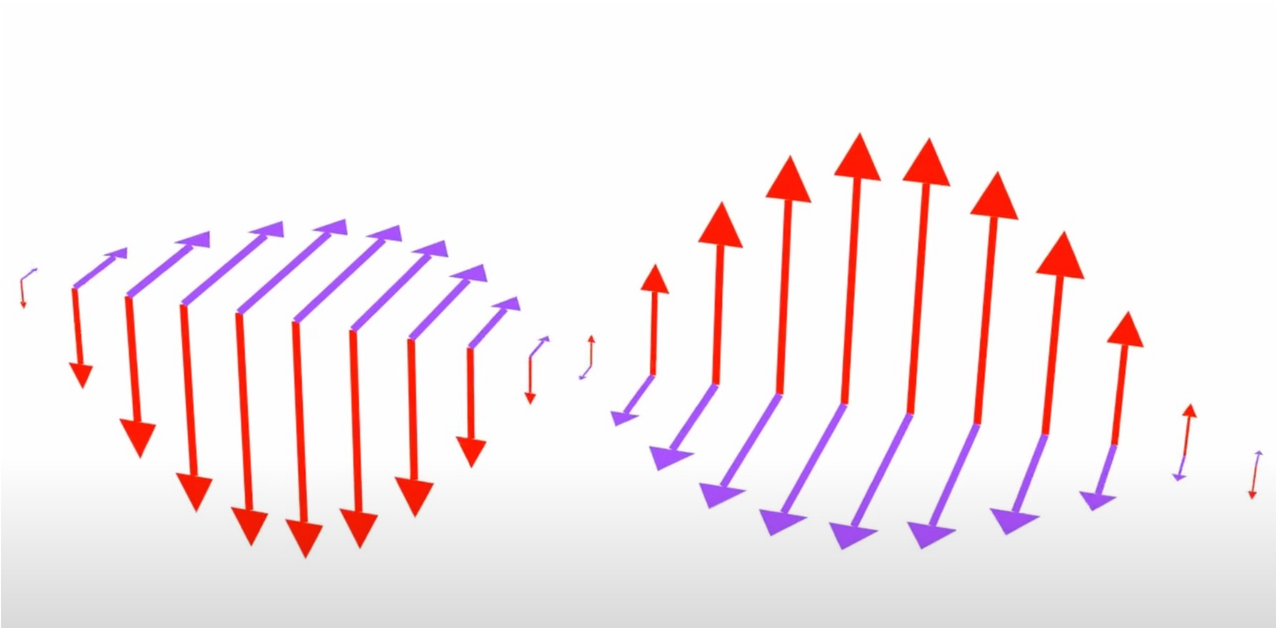
Reading

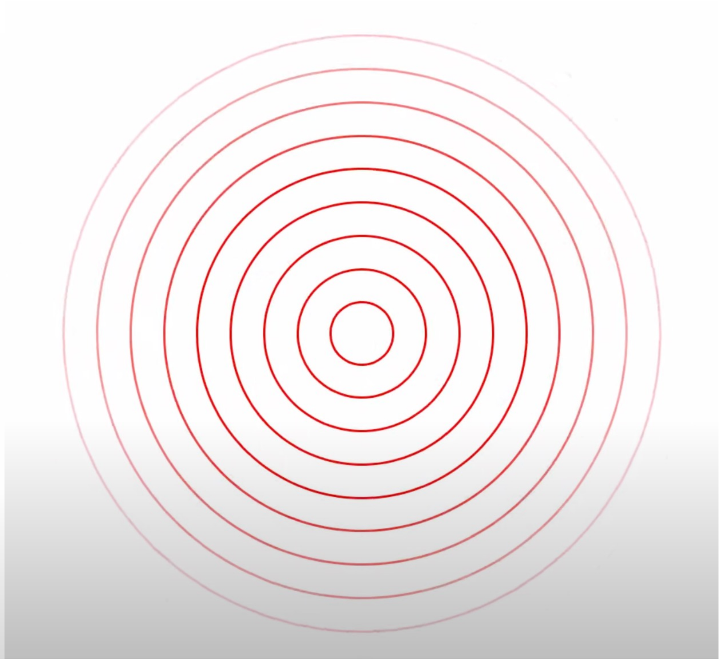
- Electromagnetic waves https://www.youtube.com/watch?v=wPdeF2EnwuM

- Wiki
  - Path loss
  - Capacity
  - SNR

Electromagnetic (EM) Waves



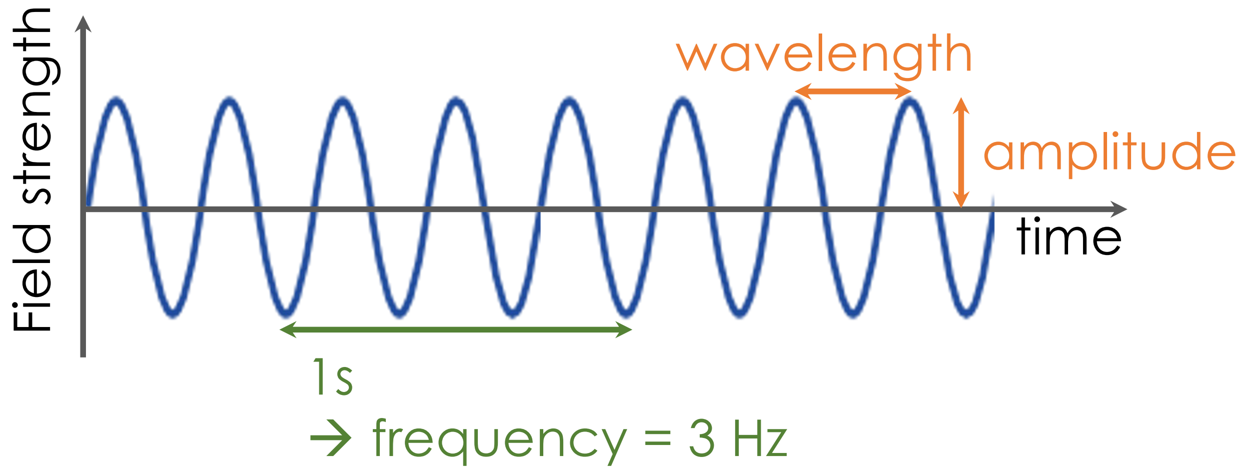
Electric field and magnetic field



2D radiation

Traverse at the light speed (3e8)

Electromagnetic Radiation



- **Amplitude**: maximum field strength of the electric and magnetic fields
- **Power**: square of the amplitude, representing the rate at which energy is transferred
- **Frequency**: number of cycles per second
- **Phase**: position of the wave in its cycle

Wireless Signals (Tx)

- Baseband signals

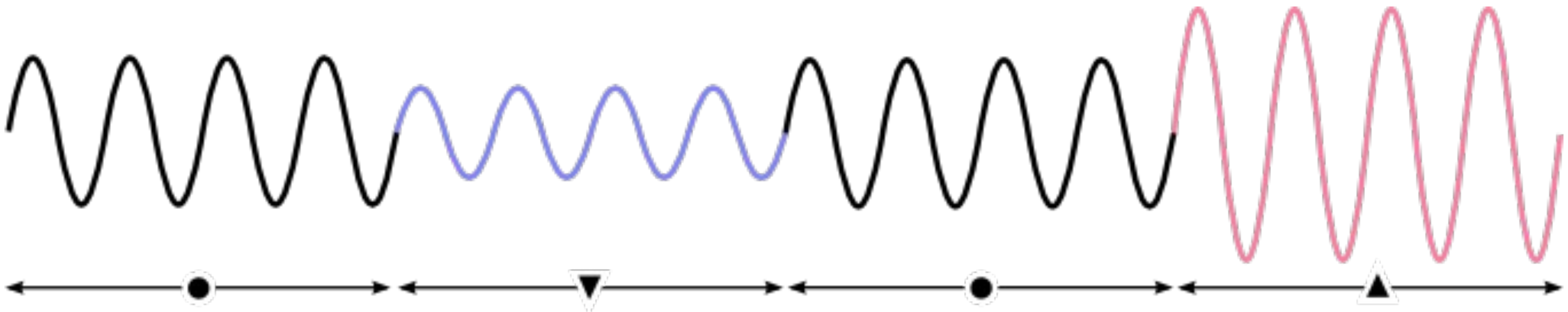
$$s(t) = a(t)cos(2\pi f_c t + \phi(t))$$

amplitude frequency phase

- Orthogonal signals

$exp(i\theta) = cos(\theta)+jsin(\theta)$

$$s'(t) = s_I(t) + js_Q(t)$$



- Transmitted signals
  - P_{tx}: transmit power (Watt)

$$s = \sqrt{P_{tx}}x$$

Wireless Signals (Rx)

- Received signals

channel noise

$$y = hx + n$$

received signal transmitted signal

- Wireless channels (channel state information, CSI)

$$h = \alpha e^{-2j\pi f_c(t+\theta)}$$

amplitude phase

Wireless Channels

$$h = \alpha e^{-2j\pi f_c(t+\theta)}$$

- **Channel state information, CSI**

- Signal variation over the air
  - Amplitude decreases with distance
    Received signal power $P_{rx} = \|h\|^2 = \|\alpha\|^2$
  - Phase change due to propagation delay
    Phase of received signal $\angle h = \theta$

Signal Power

- Watt vs. Decibel (**dBm or dB_{mW}**)
  - dBm is usually used in radio
  - Converted from milliwatt
  - Express both very large and small values in a short form

$$P_{dBm} = 10\log_{10}(1000 P_W) \qquad P_W = \frac{10^{P_{dBm}/10}}{1000}$$

- **dB**: the **difference** between two dBm values
  - ratio of two power = difference between two dBm

$$\text{P1toP2}_{dB} = 10\log_{10}\left(\frac{P_1}{P_2}\right)$$
$$= 10\log_{10}(P_1) - 10\log_{10}(P_2)$$
$$= P_{1,dBm} - P_{2,dBm}$$

Power vs. dB

- Because of the log operation, double the power produces 3dB gain

$$SNR_{dB} = 10\log_{10} SNR$$

- **3dB gain**

$$P_1 = 2 * P_2$$

$$\Longrightarrow 10\log_{10}\frac{P_1}{N} = 10\log_{10}\frac{2 * P_2}{N}$$

$$P_{1,dB} = P_{2,dB} + 10\log_{10} 2 = P_{2,dB} + 3.0103(\mathrm{dB})$$

Path Loss

- Attenuation reduction as the signal propagates through the air
- **Friis Transmission Formula**

$$\frac{P_r}{P_t} = D_t D_r \left(\frac{\lambda}{4\pi d}\right)^2 \quad \text{(in Watt)}$$

$$P_r - P_t = D_t + D_r + 20\log_{10}\left(\frac{\lambda}{4\pi d}\right) \quad \text{(in dB)}$$

  - λ: signal wavelength
  - P_t/P_r: transmitting/receiving power
  - D_t/D_r: Gains of transmitting/receiving antennas
- Loss ∝ distance^2

Shannon Capacity

- The tight **upper bound** on the data rate

$$C = B\log_2\left(1+\frac{S}{N}\right) = B\log_2(1+SNR)$$

- B: **bandwidth** (Hz), e.g., WiFi with 20MHz
- S and N is in **Watt** (SNR is power ratio, not in dB)
- Example: SNR=25dB, what is the capacity of 20MHz WiFi?

$$SNR_{dB} = 10 * \log_{10} SNR \Rightarrow SNR = 10^{SNR_{dB}/10} = 316.2278$$

$$C = 20 * 10^6 * \log_2(1+316.2278) = 166.1875(\mathrm{Mbps})$$

Shannon Capacity

- In low SNR regime, increasing SNR can increase the rate significantly
- In high SNR regime, the increase in rate from SNR gain is relatively small

- 4dB → 7dB
  - SNR: 2.5119 → 5.0119
  - Capacity: 1.8123 → 2.5878 (1.43x) big enhancement
- 30dB → 33dB
  - SNR: 1000 → 1.9953e+03
  - Capacity: 9.9672 → 10.9631 (1.0999x) small enhancement