Network System Capstone
@CS.NYCU

2025.02.27: Analog Beamforming

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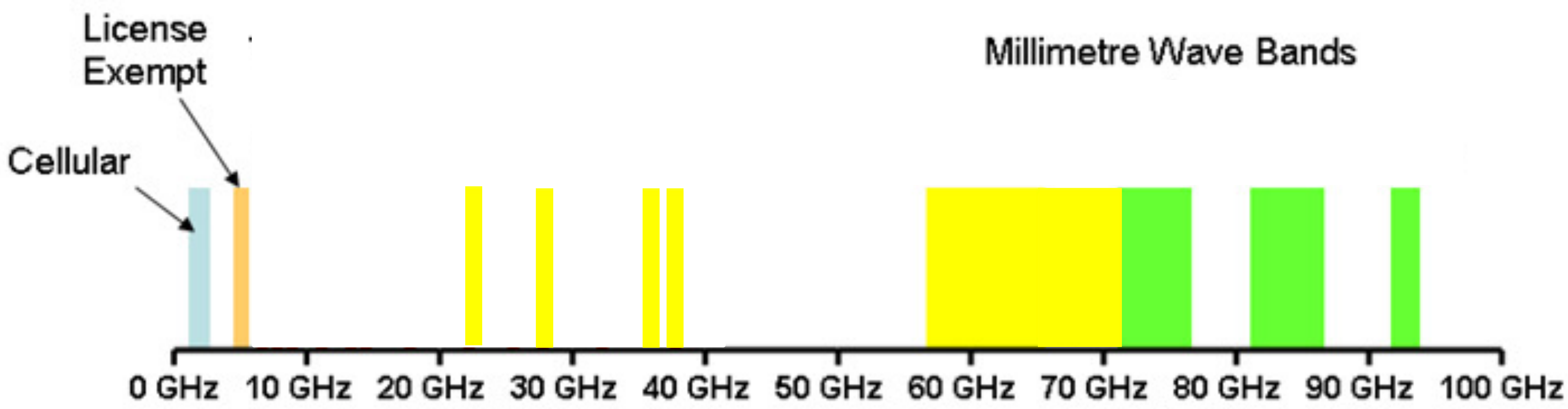
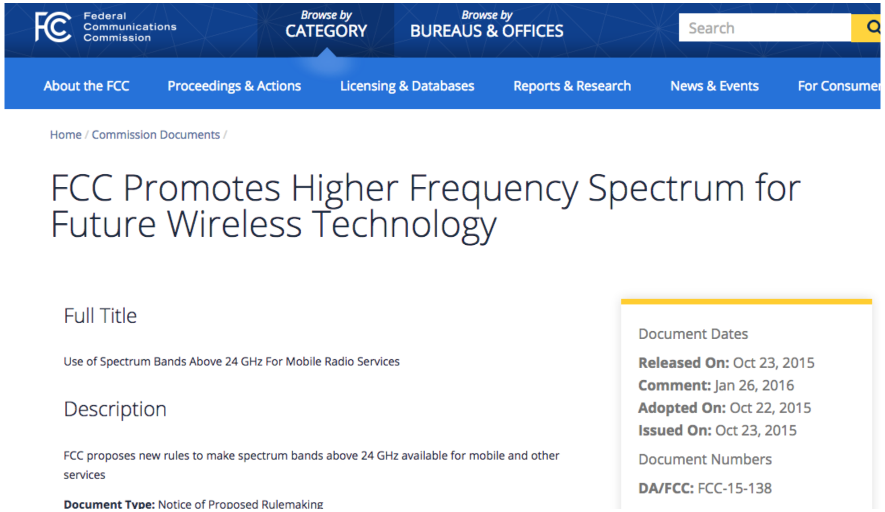
Millimeter Wave Bands

- Huge amount of available bandwidth ($\lambda = C/f$)





FCC Promotes Higher Frequency Spectrum for Future Wireless Technology

Full Title

Use of Spectrum Bands Above 24 GHz For Mobile Radio Services

Description

FCC proposes new rules to make spectrum bands above 24 GHz available for mobile and other services

Document Type: Notice of Proposed Rulemaking

Document Dates

Released On: Oct 23, 2015
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Issued On: Oct 23, 2015

Document Numbers

DA/FCC: FCC-15-138

Advanced Wireless Research Initiative @ NSF

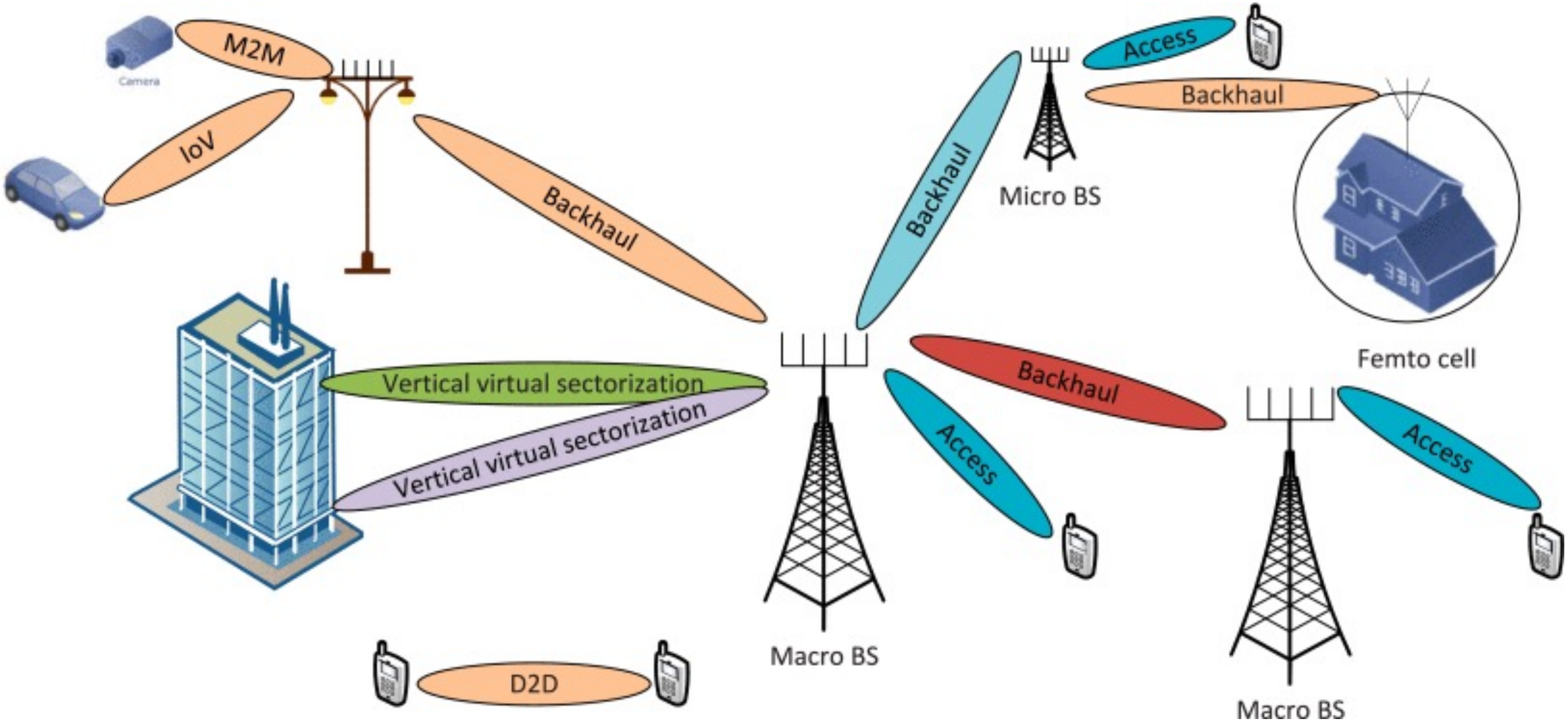
The Advanced Wireless Research Initiative will sustain United States leadership in wireless communications and tech
and development.

The National Science Foundation's (NSF) leadership of this Initiative has three intertwined components:

- Establishing **platforms for advanced wireless research** enabled by a new industry consortium and engagem
- Supporting **fundamental research enabling advanced wireless technologies**; and
- Catalyzing **academic, industry, and community leaders** to work together to prototype innovative wireless app

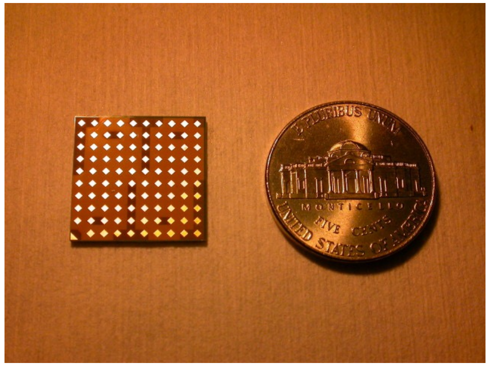
These efforts will provide new insights capable of making wireless communication faster, smarter, more responsive, a

mmWave Massive MIMO Beamforming

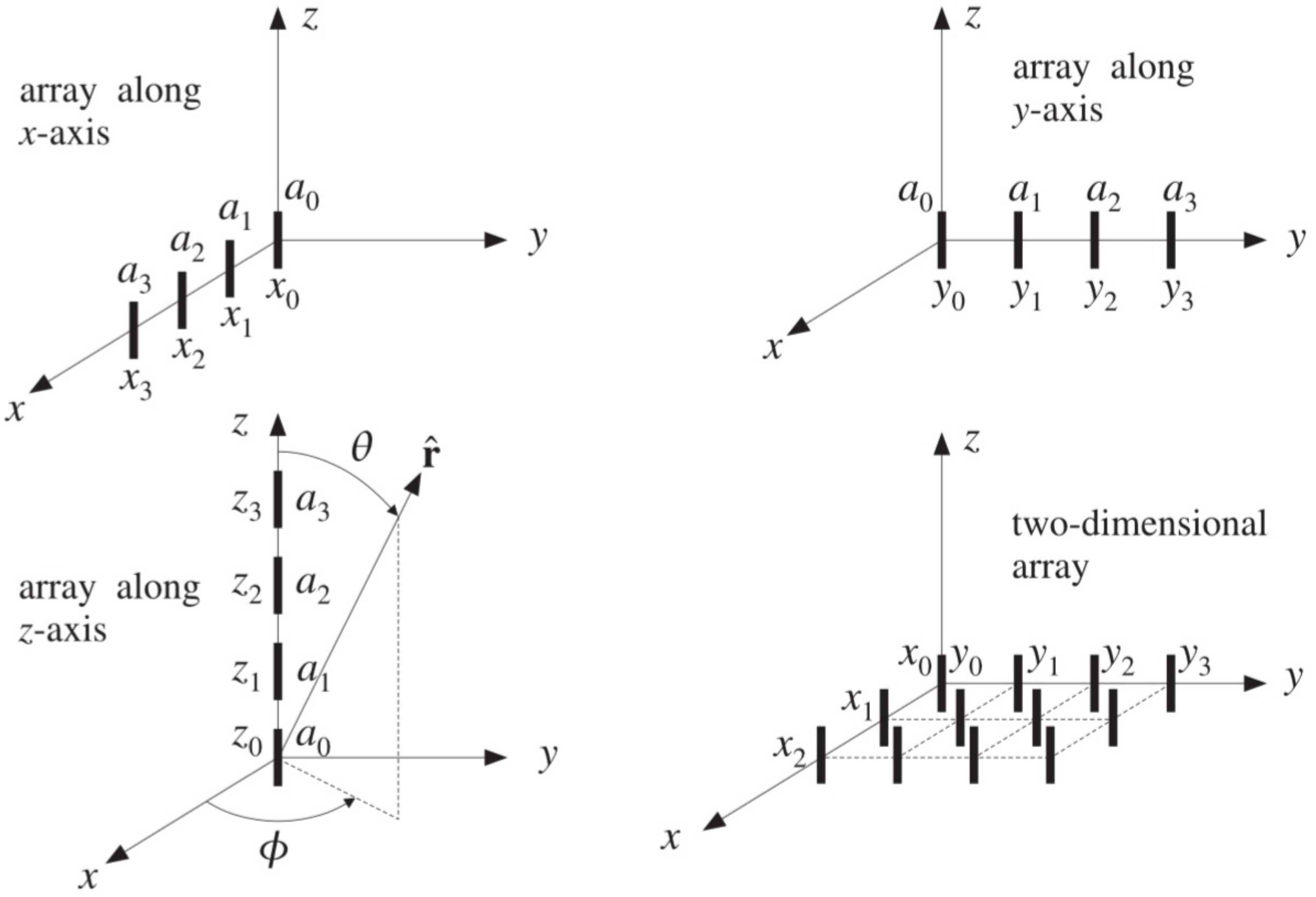


Beam Steering

- Direct radiated power towards a desired angular sector
- Does not need to know the channel state information
- How? Phase array
  - By changing the phase of each antenna
  - Also known as switched-beam antenna or adaptive antenna
- Beam pattern is determined by
  - the number of antennas
  - the arrangement of antennas



Array Configuration

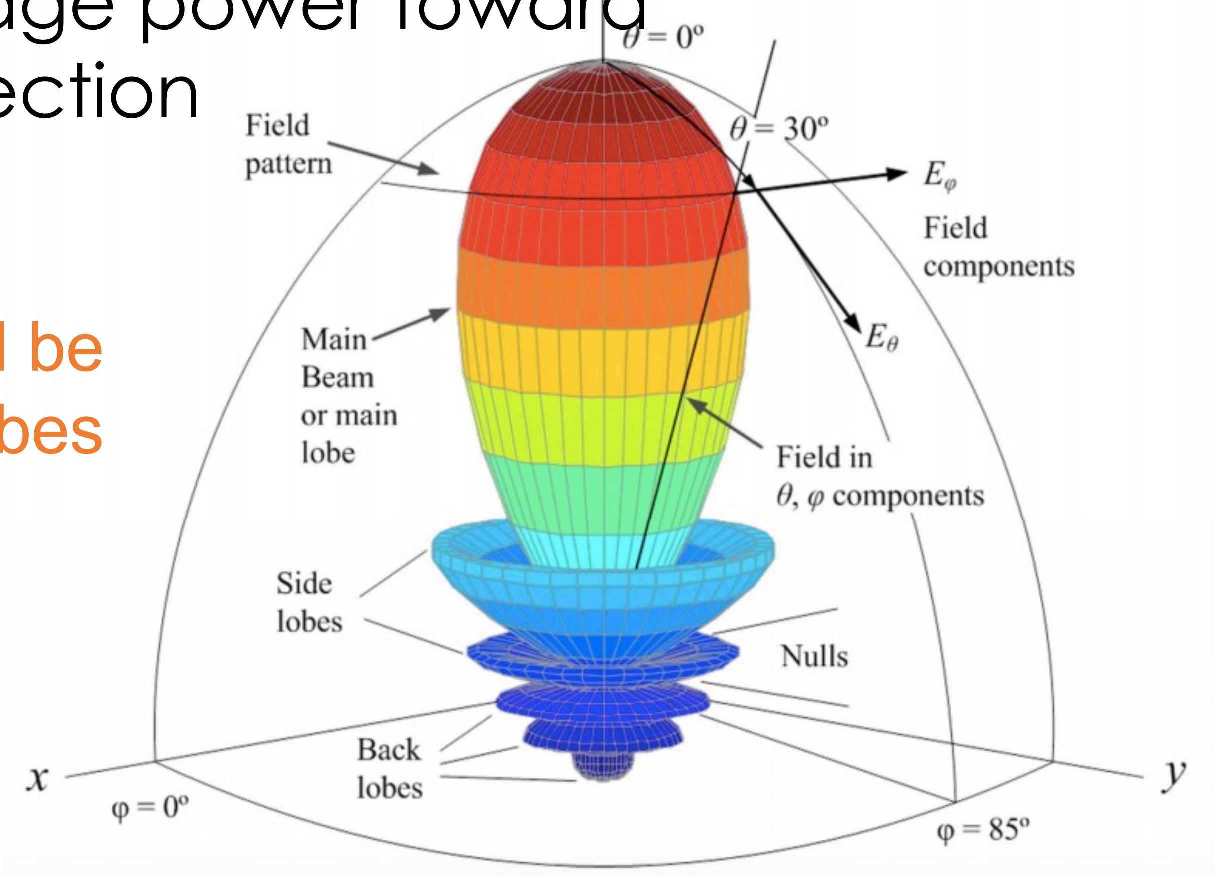


Can be 1D or 2D

Main lobe and Side lobe

- Main lobe: the beam with the strongest power
- Side lobe: leakage power toward undesirable direction

There could be multiple side lobes

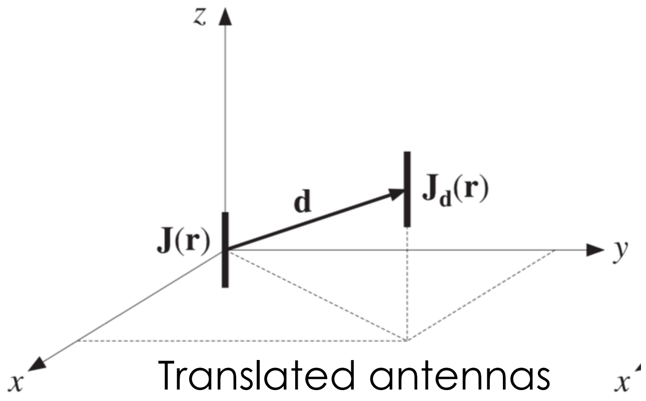


Reference: https://www.youtube.com/watch?v=V8bqxBywvjc

Translational Phase Shift

- Relative displacements of the antenna elements with respect to each other introduce relative phase shifts in the radiation vectors
- Current density of the translated antenna:

$$J_d(r) = J(r - d)$$



Translated antennas by vector d

Translational Phase Shift

- Translation in space or time domain
  → phase shift in the Fourier domain
- Radiation vector is the three-dimensional Fourier transform of the current density

$$\mathbf{F_d} = \int e^{j\mathbf{k}\cdot\mathbf{r}}\,\boxed{\mathbf{J_d}(\mathbf{r})}\,d^3\mathbf{r} = \int e^{j\mathbf{k}\cdot\mathbf{r}}\,\mathbf{J}(\mathbf{r}-\mathbf{d})\,d^3\mathbf{r} = \int e^{j\mathbf{k}\cdot(\mathbf{r}'+\mathbf{d})}\,\mathbf{J}(\mathbf{r}')\,d^3\mathbf{r}'$$

$$= e^{j\mathbf{k}\cdot\mathbf{d}}\int e^{j\mathbf{k}\cdot\mathbf{r}'}\,\mathbf{J}(\mathbf{r}')\,d^3\mathbf{r}' = \boxed{e^{j\mathbf{k}\cdot\mathbf{d}}}\,\mathbf{F}$$

$$\boxed{F(w) = \int_{-\infty}^{\infty} f(x)\,e^{-iwx}\,dx}$$

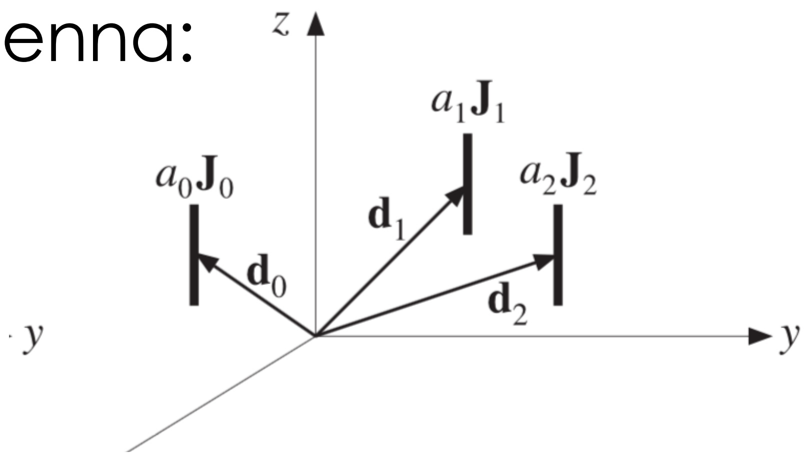
Fourier transformation

- By changing variables to r′ = r − d, we get translation phase shift

$$\boxed{\mathbf{F_d}(\mathbf{k}) = e^{j\mathbf{k}\cdot\mathbf{d}}\,\mathbf{F}(\mathbf{k})}$$

Array Pattern Multiplication

- Consider a three-dimensional array of several identical antennas located at positions d_0, d_1, d_2, … with relative feed coefficients a_0, a_1, a_2, …
- Current density of the nth antenna:
  $J_n(r) = a_n J(r - d_n)$



$$\boldsymbol{F}_n(\boldsymbol{k}) = a_n e^{j\boldsymbol{k}\cdot\mathbf{d}_n}\boldsymbol{F}(\boldsymbol{k})$$ Radiation vector

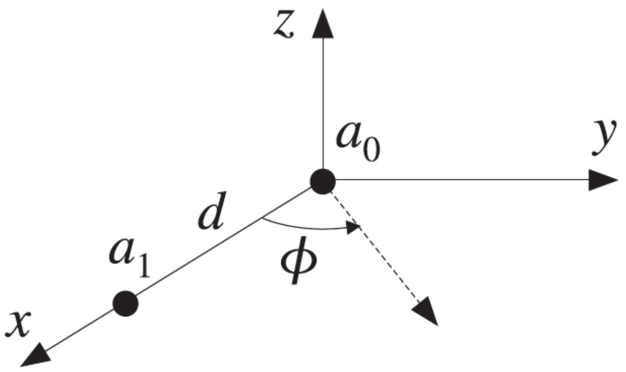
$$\boldsymbol{J}_{\text{tot}}(\boldsymbol{r}) = a_0\boldsymbol{J}(\boldsymbol{r}-\mathbf{d}_0) + a_1\boldsymbol{J}(\boldsymbol{r}-\mathbf{d}_1) + a_2\boldsymbol{J}(\boldsymbol{r}-\mathbf{d}_2) + \cdots$$ Total current density

$$\boldsymbol{F}_{\text{tot}}(\boldsymbol{k}) = \boldsymbol{F}_0 + \boldsymbol{F}_1 + \boldsymbol{F}_2 + \cdots = a_0 e^{j\boldsymbol{k}\cdot\mathbf{d}_0}\boldsymbol{F}(\boldsymbol{k}) + a_1 e^{j\boldsymbol{k}\cdot\mathbf{d}_1}\boldsymbol{F}(\boldsymbol{k}) + a_2 e^{j\boldsymbol{k}\cdot\mathbf{d}_2}\boldsymbol{F}(\boldsymbol{k}) + \cdots$$

$$\boldsymbol{F}_{\text{tot}}(\boldsymbol{k}) = A(\boldsymbol{k})\boldsymbol{F}(\boldsymbol{k})$$ Total radiation vector

$$A(\boldsymbol{k}) = a_0 e^{j\boldsymbol{k}\cdot\mathbf{d}_0} + a_1 e^{j\boldsymbol{k}\cdot\mathbf{d}_1} + a_2 e^{j\boldsymbol{k}\cdot\mathbf{d}_2} + \cdots$$ Array vector

Example: two antennas at [0, d]



- At polar angle θ = 90°, that is, on the xy-plane, the array factor will be:

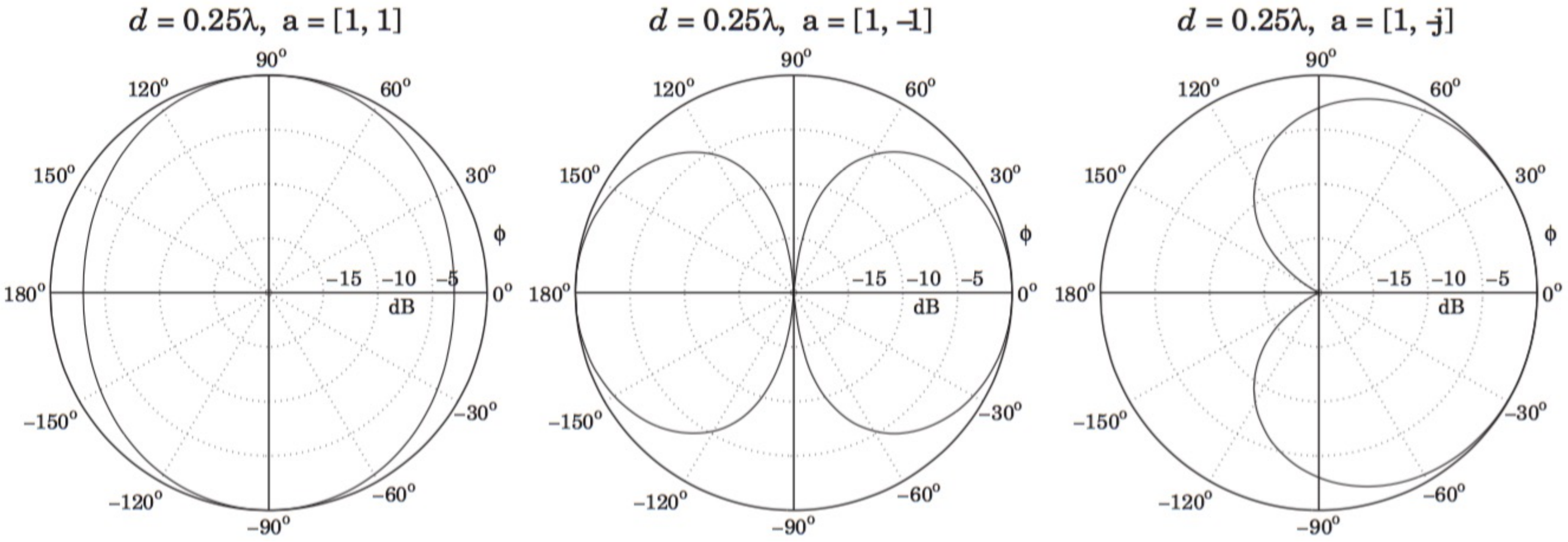
$$A(\phi) = a_0 + a_1 e^{jkd\cos\phi}$$

- The azimuthal power pattern:

$$g(\phi) = |A(\phi)|^2 = \left|a_0 + a_1 e^{jkd\cos\phi}\right|^2$$

Example: two antennas at [0, d]

- MATLAB: `[g, phi] = gain1d(d, a, 400);`

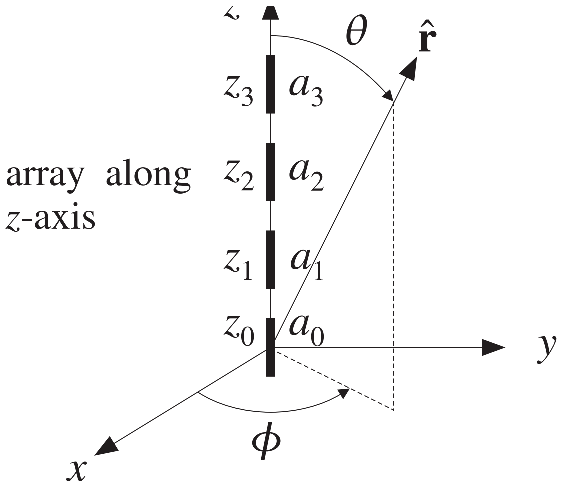


Azimuthal gain patterns A(φ) of different array weights $[a_0, a_1]$

Uniform Array

- Sum of the weights is unity

$$\mathbf{a} = [a_0, a_1, \ldots, a_{N-1}] = \frac{1}{N}[1, 1, \ldots, 1]$$



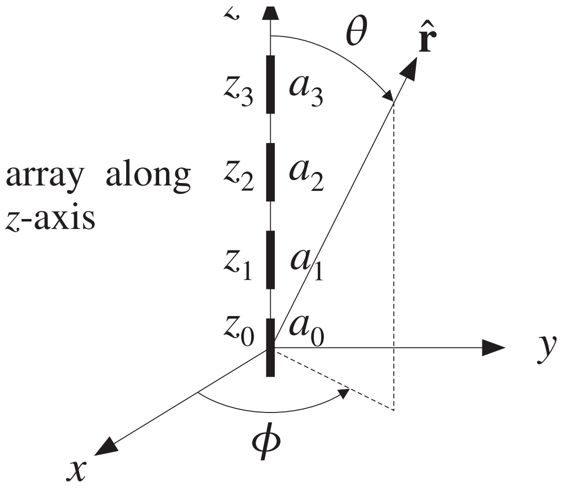
- Array factor

$$A(\psi) = \frac{1}{N}\left[1 + e^{j\psi} + e^{2j\psi} + \cdots + e^{(N-1)j\psi}\right] = \frac{1}{N}\,\frac{e^{jN\psi} - 1}{e^{j\psi} - 1}$$

$$g(\phi) = |A(\psi)|^2$$

Uniform Array

- φ: Azimuth angle
- θ: Zenith angle
- G(θ, φ): power gain of a signal toward (θ, φ)



- How to manipulate the gain?
  - Change the phase of each antenna via a phase shift a_i

$h_1 + h_2 + \cdots + h_N$ Original channel

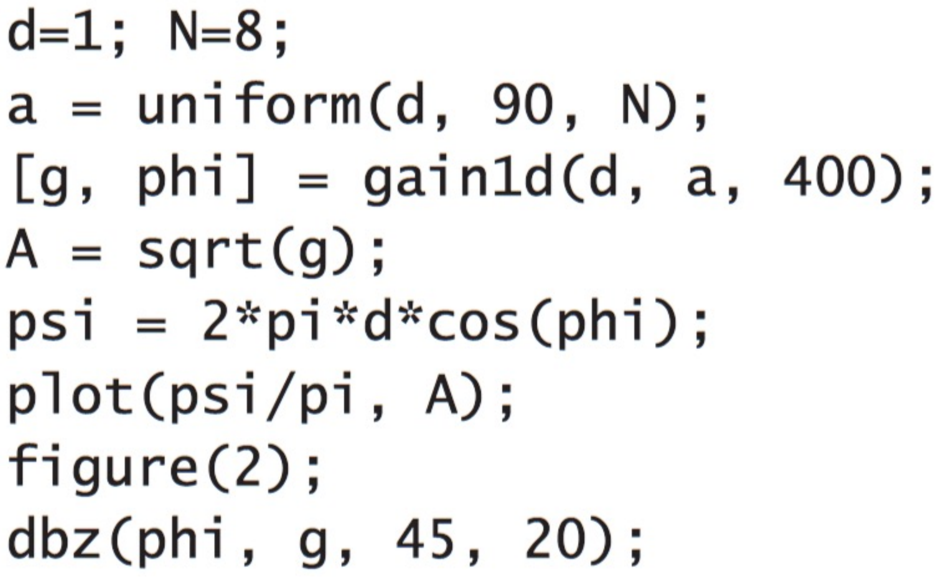
$a_1 h_1 + a_1 h_2 + \cdots + a_N h_N$ Channel of steering signals

$$\boxed{h = e^{-2j\pi ft} \leftrightarrow ah = e^{-2j\pi ft} * e^{\delta} = e^{-2j\pi ft+\delta}}$$

G(θ, φ) is independent of the original channel *h*

Uniform Array

- How to get coefficient a_n for a desired direction?



```
d=1; N=8;
a = uniform(d, 90, N);
[g, phi] = gain1d(d, a, 400);
A = sqrt(g);
psi = 2*pi*d*cos(phi);
plot(psi/pi, A);
figure(2);
dbz(phi, g, 45, 20);
```

Array Factor of Uniform Array

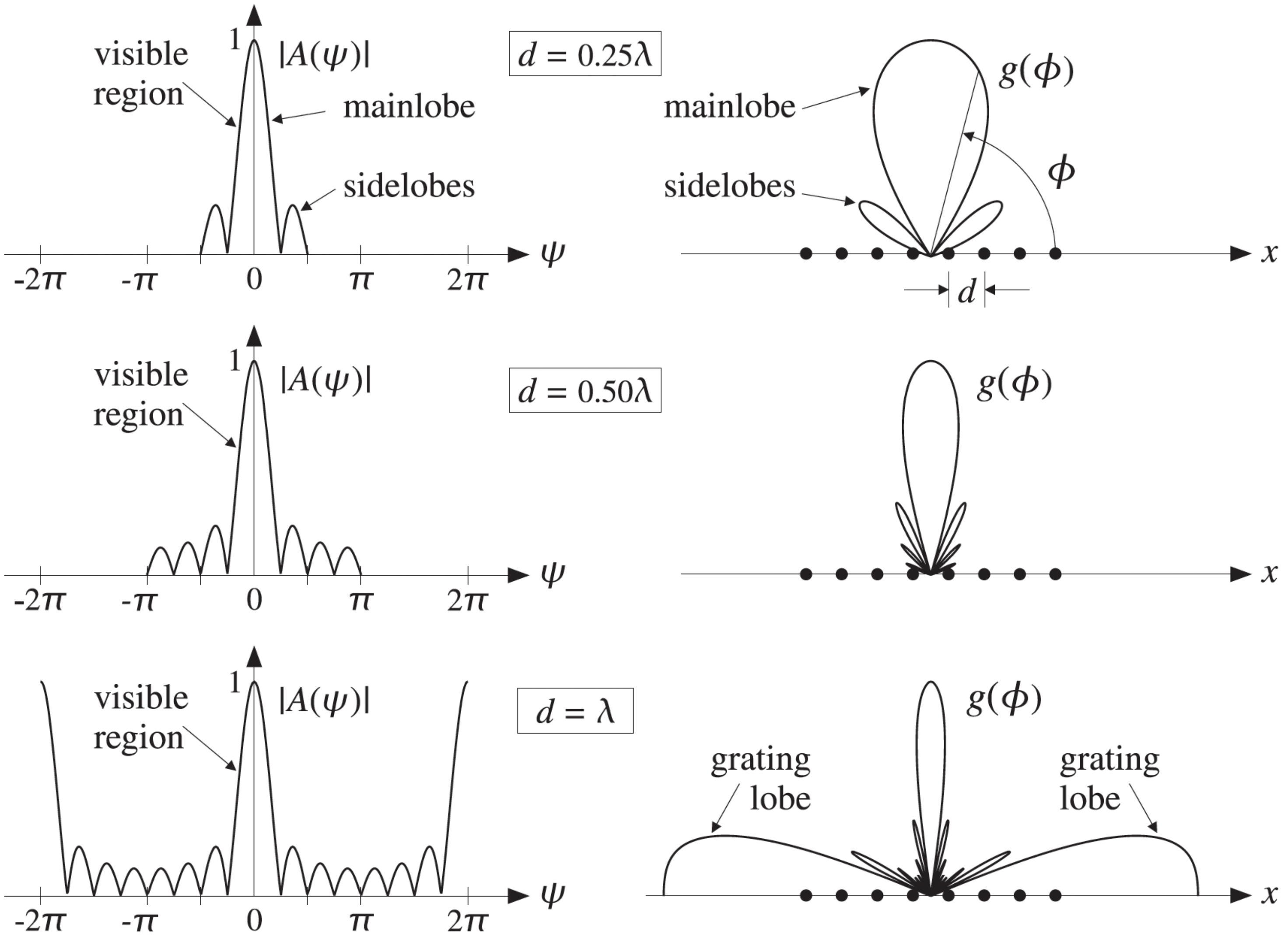


Fig. 22.7.1 Array factor and angular pattern of 8-element uniform array.

Beamwidth of Uniform Array

- 3dB width: half of the peak gain
- Beamwidth gets narrower with increasing N
  - – Intuition: more array factors *a* we can control to get a better resolution

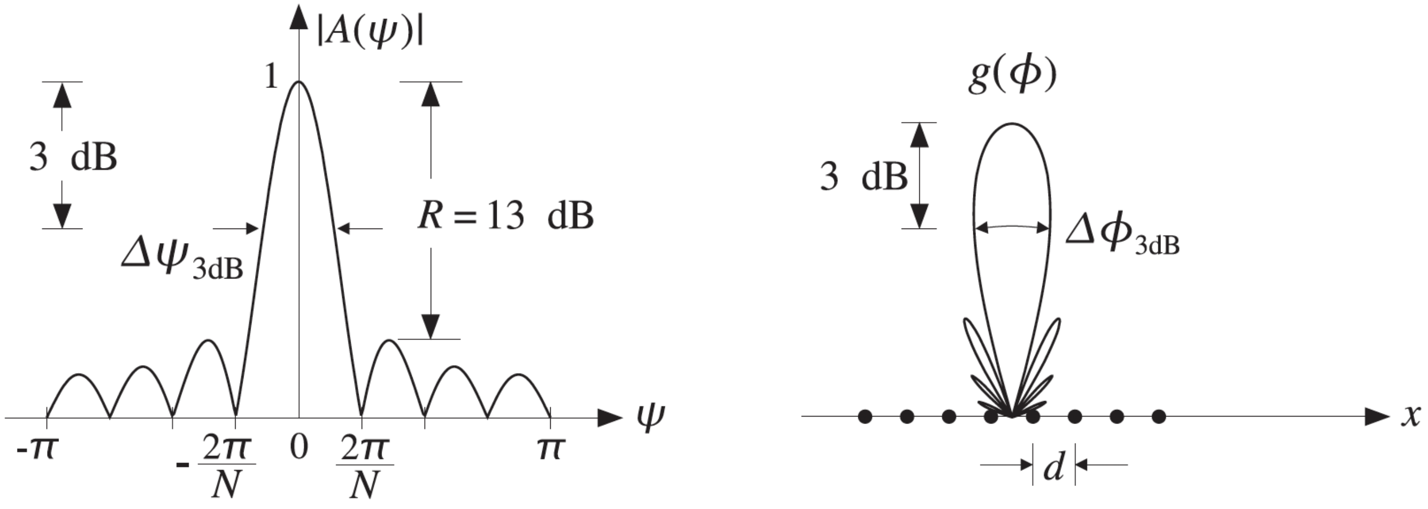
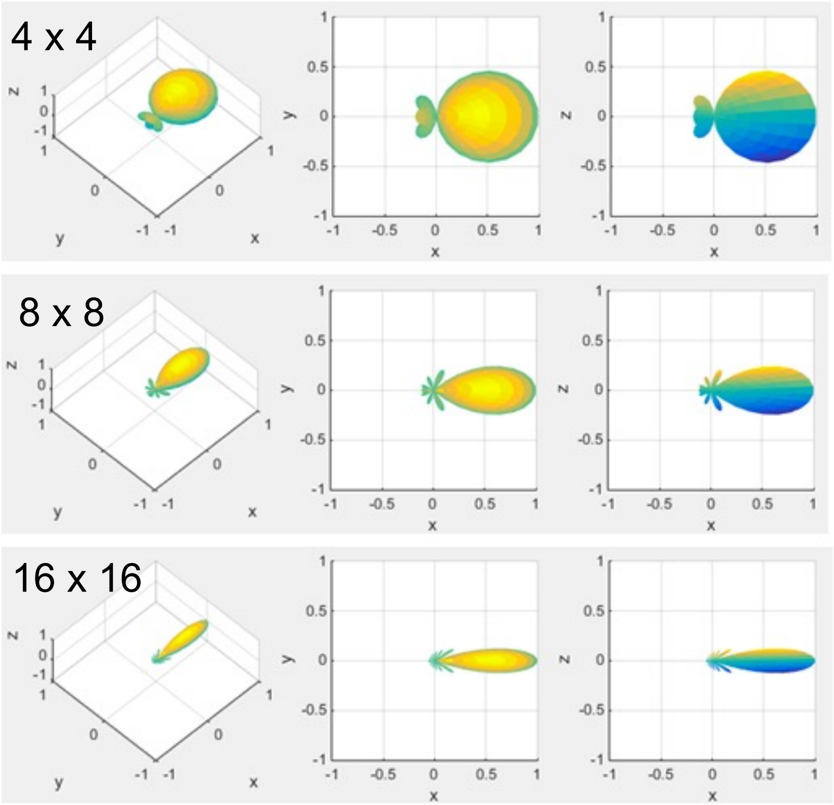


Fig. 22.7.3 Mainlobe width and sidelobe level of uniform array.

$$\Delta\psi_{3\text{dB}} = \left|\frac{\partial\psi}{\partial\phi}\right| \Delta\phi_{3\text{dB}} = kd\,\Delta\phi_{3\text{dB}}$$

$$\Delta\phi_{3\text{dB}} = 0.886\,\frac{\lambda}{Nd}$$

Beamwidth of Uniform Array



Array Directivity

- Directivity of the array:

$$D = \frac{4\pi}{\Delta\Omega} = \frac{\left|\sum_n a_n\right|^2}{\sum_{n,m} a_n a_m^* \frac{\sin(kd(n-m))}{kd(n-m)}}$$

- The uniform array with half-wavelength spacing achieves *maximum directivity* equal to the number of array elements
  - Optimum array vector a = $[a_0, a_1, \ldots, a_{N-1}]^T$ that maximizes D:

$$\mathbf{a} = A^{-1}\mathbf{u}$$

corresponding maximum directivity:

$$D_{\max} = \mathbf{u}^T A^{-1}\mathbf{u}$$

where u = $[1,1,\ldots,1]^T$

Array Directivity

- Matrix form

$$A_{nm} = \frac{\sin\left(kd\left(n-m\right)\right)}{kd\left(n-m\right)}, \quad 0 \le n, m \le N-1$$

Array Steering

- At φ=90°, the maximam of the array factor $A(\psi)$ corresponds to $\psi=kd\cos\varphi$
  - $|A|_{max} = |A(0)|$

$$\boxed{\psi_0 = kd\cos\phi_0} \qquad \text{(steering phase)}$$

- Array pattern towards some other direction φ_0

$$\boxed{\psi_0 = kd\cos\phi_0} \qquad \text{(steering phase)}$$

$$\boxed{A'(\psi) = A(\psi - \psi_0)} \qquad \text{(steered array factor)}$$

Array Steering

- Change the direction of the main lobe by changing the array factor A
- Usually, A can only be selected from a fixed-sized codebook (limited number of directions)