National Chiao Tung University, Department of Computer Science
DCP 3351 Probability Theory–Mid-term II

Instructor: Dr. Wen-Hsiao Peng
Date: Tuesday, Dec. 16, 2014
Time: 1:20pm – 3:10pm (110 minutes)
Format: Closed book

$\frac{f_{X,Y}(x,y)}{f_Y(y)} = f_{X|Y}(x|y)$

Instructions:
1) You may give your answers in Chinese or English.
2) Please give your answers in succinct phrases or point form.
3) Please write your answers clearly (with explicit denotation of labels and symbols used).

1. (20 pts) A sufficient number of voters are polled to determine the percentage in favor of a certain candidate. Assuming that an unknown proportion p of the voters favor him and they act independently of one another, how many should be polled to predict the value of p within 0.045 with 95% confidence?

   (a) Evaluate the answer using Chebyshev inequality, $P(|X - \mu_X| \geq c) \leq \frac{\sigma^2}{c^2}$.

   (b) Evaluate the answer using the Central Limit Theorem.

$E[X^2]$ μ^2

μ σ

2. (40 pts) Assume X, Y are two correlated random variables with $X \sim U(0, 1)$ and $f_{Y|X}(y|X = x) \sim U(x - 1, x + 0)$.

   cov

   (a) Compute the PDF, mean and variance of Y.

   $E[X]E[Y]$ $= E[(X-\mu)(Y-\mu)]$

   (b) Compute the mean and variance of $Z = 2X - 3Y + 1$.

   (c) Compute the correlation coefficient $\rho = Cov(Z, Y)/\sigma_Z\sigma_Y$ between Z and Y.

   $= E[XY]$

   (d) Compute the conditional PDF $p(x|Z = z)$ and the mean $E(X|Z = z)$.

   $- E[X]E$

3. (30 pts) Assume X_1 and X_2 are two independent random variables with geometric distribution. Their parameters are p_1 and p_2, respectively.

$$P_{X_1}(k) = (1 - p_1)^{k-1} p_1, k = 1, 2, 3, ...$$
$$E[X_1] = \frac{1}{p_1}, var(X_1) = \frac{1 - p_1}{p_1^2}$$

$f_X = \begin{cases} \lambda e^{-\lambda} & x \geq 0 \\ & \text{oth} \end{cases}$

$E X + E Y$

probility Mass Function.

   (a) Compute the PMF of $Y = \min(X_1, X_2)$.

   $f_X = \lambda e^{-\lambda}$

   (b) Compute the mean and the variance of Y.

   $E = \frac{1}{\lambda}$

   (c) Compute the conditional mean of $Y - 5$ given $Y > 5$, $E[Y - 5|Y > 5]$.

   $Var = \frac{1}{\lambda^2}$

4. (20 pts) Let X_1, X_2 and X_3 be *independent* and *identical* binomial random variables $B(n; p)$, that is, $P(X_i = k) = \binom{n}{k} p^k (1 - p)^{n-k}$, $0 \leq k \leq n$.

   (a) Compute the probability distribution of $Z = X_1 + X_2 + X_3$.

   (b) Compute $E(Z^3)$.

$E(Z^2)$

$E[Z]$

$0 \leq t \leq 3n$

3. $Var(Z) = 3np(1-p)$