Probability: Homework Set One, February 24, 2026

Due: March 10, 2026

$C - C\cap A - C\cap B + A\cap B\cap C$

$= C\cap(1-A-B+A\cap B) = A^c - B\cap A^c$

$= C\cap$

1) We have known that $\mathbf{P}(A\cup B) = \mathbf{P}(A) + \mathbf{P}(A^c \cap B)$. Prove $\mathbf{P}(A\cup B\cup C) = \mathbf{P}(A) + \mathbf{P}(A^c \cap B) + \mathbf{P}(A^c \cap B^c \cap C)$. Note that you are **not allowed** to use the Venn Diagram.
2) Consider an experiment whose sample space consists of a countably infinite number of points.
   a) Show that not all points can be equally likely.
   b) Can all points have a positive probability of occurring? Prove your answer or give an example.
3) A conventional knock-out tournament begins with 2^n competitors and has n rounds. There are no play-offs for the positions 2, 3, $\cdots$, $2^n - 1$, and the initial table of draws is specified. Give a concise description of the sample space of all possible outcomes.
4) Describe the sample spaces for the following experiments. You need to use H as head and T as tail.
   a) A biased coin is tossed three times.
   b) A biased coin is tossed repeatedly until a head turns up.
5) Let A and B be events with probabilities $\mathbf{P}(A) = \frac{3}{4}$ and $\mathbf{P}(B) = \frac{1}{3}$. Show that $\frac{1}{12} \leq \mathbf{P}(A\cap B) \leq \frac{1}{3}$, and give examples to show that both extremes are possible. Find corresponding bounds for $\mathbf{P}(A\cup B)$.
6) Out of the students in a class, 60% love comics, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a comic lover nor a chocolate lover.
7) Problems 3, 11, 12, and 13 from the textbook. **Note that these questions are for practice, review, or optional study purposes and do not need to be submitted.**

1. $P(A\cup B\cup C) = P((A\cup B)\cup C) = P(A\cup B) + P((A\cup B)^c \cap C)$
$= P(A) + P(A^c\cap B) + P(A^c \cap B^c \cap C)$

2. (a) assume all $s_i \in S$ have same $P(s_i)=P$, then

$\begin{cases} \text{if } P>0 \Rightarrow \sum_{i=1}^{\infty} P(s_i) = \infty \neq 1 \\ \text{if } P=0 \Rightarrow \sum_{i=1}^{\infty} P(s_i) = 0 \neq 1 \end{cases} \Rightarrow P(s_i)$ can't be a constant

(b) yes, let $P(s_i) = \left(\frac{1}{2}\right)^i \Rightarrow \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i = 1 = P(S)$

3. $2^n - 1$ competitions $\Rightarrow$ for each $w\in S$, w is length 2^n-1 construct by winner of each match

4. (a) $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

(b) $S = \{T^k H \mid k = \{0,1,2,\dots\}\}$

5. $\begin{cases} 1 \geq P(A\cup B) = P(A)+P(B)-P(A\cap B) \Rightarrow P(A\cap B) \geq \frac{1}{12} \Rightarrow \text{if } A\cap B = S \\ P(A\cap B) \leq P(A) = \frac{3}{4} \\ P(A\cap B) \leq P(B) = \frac{1}{3} \Rightarrow \text{if } B\subset A \end{cases} \Rightarrow \frac{1}{12} \leq P(A\cap B) \leq \frac{1}{3}$

$P(A\cup B)$ have max 1 when $P(A\cap B)$ have min since $P(A\cup B) = P(A)+P(B)-P(A\cap B)$
$P(A\cup B)$ have min $\frac{3}{4}$ when $P(A\cap B)$ have max value

6. $P(A) = 0.6 \Rightarrow$ love comic
$P(B) = 0.7 \Rightarrow$ love chocolate
$P(A\cap B) = 0.4$
$P(A\cup B) = 0.6+0.7-0.4 = 0.9$

$P(A^c \cap B^c) = P((A\cup B)^c) = 1 - P(A\cup B) = 0.1$