Probability: Homework Set One, February 24, 2026
Due: March 10, 2026

1) We have known that $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(A^c \cap B)$. Prove $\mathbf{P}(A \cup B \cup C) = \mathbf{P}(A) + \mathbf{P}(A^c \cap B) + \mathbf{P}(A^c \cap B^c \cap C)$. Note that you are **not allowed** to use the Venn Diagram.
2) Consider an experiment whose sample space consists of a countably infinite number of points.
   a) Show that not all points can be equally likely.
   b) Can all points have a positive probability of occurring? Prove your answer or give an example.
3) A conventional knock-out tournament begins with 2^n competitors and has n rounds. There are no play-offs for the positions 2, 3, $\cdots$, $2^n - 1$, and the initial table of draws is specified. Give a concise description of the sample space of all possible outcomes.
4) Describe the sample spaces for the following experiments. You need to use H as head and T as tail.
   a) A biased coin is tossed three times.
   b) A biased coin is tossed repeatedly until a head turns up.
5) Let A and B be events with probabilities $\mathbf{P}(A) = \frac{3}{4}$ and $\mathbf{P}(B) = \frac{1}{3}$. Show that $\frac{1}{12} \leq \mathbf{P}(A \cap B) \leq \frac{1}{3}$, and give examples to show that both extremes are possible. Find corresponding bounds for $\mathbf{P}(A \cup B)$.
6) Out of the students in a class, 60% love comics, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a comic lover nor a chocolate lover.
7) Problems 3, 11, 12, and 13 from the textbook. **Note that these questions are for practice, review, or optional study purposes and do not need to be submitted.**