Probability: Homework Set Two, March 4, 2026
Due: March 20, 2026

1) Prove *Boole's inequalities*:
   a) $\mathbf{P}\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} \mathbf{P}(A_i)$,
   b) $\mathbf{P}\left(\bigcap_{i=1}^{n} A_i\right) \geq 1 - \sum_{i=1}^{n} \mathbf{P}(A_i^c)$.
2) On rainy days, Joe is late to work with probability 0.3; on nonrainy days, he is late with probability 0.1. With probability 0.7, it will rain tomorrow.
   a) Find the probability that Joe is not late tomorrow.
   b) Given that Joe was not late, what is the conditional probability that it rained?
3) A random number N of dice is thrown. A_i is the event that $N = i$, and $\mathbf{P}(A_i) = 2^{-i}$, $i \geq 1$. The sum of the scores is S. Find the probability that:
   a) $N = 2$ given $S = 4$,
   b) $S = 4$ given N is even.
4) *Polya's urn scheme*. Suppose an urn has r red balls and b black balls. A ball is drawn and its color noted. Then it together with $c > 0$ balls of the same color as the drawn ball are added to the urn. The procedure is repeated $n - 1$ additional times so that the total number of drawings made from the urn is n.
   Let R_j, $1 \leq j \leq n$, denote the event that the *j*th ball drawn is red and let B_j, $1 \leq j \leq n$, denote the event that the *j*th ball drawn is black. Answer the following questions.
   a) $\mathbf{P}(R_1)$=?
   b) $\mathbf{P}(R_2)$=?
   c) What is $\mathbf{P}(R_j)$ for $j \geq 3$? Guess it. And, then prove your answer.
   d) $\mathbf{P}(R_1|R_2)$=?
5) What do you think of the following 'proof' by Lewis that an urn cannot contain two balls of the same color?
   Suppose that the urn contains two balls, each of which is either black or white; thus, in the obvious notation, $\mathbf{P}(BB) = \mathbf{P}(BW) = \mathbf{P}(WB) = \mathbf{P}(WW) = \frac{1}{4}$. We add a black ball, so that $\mathbf{P}(BBB) = \mathbf{P}(BBW) = \mathbf{P}(BWB) = \mathbf{P}(BWW) = \frac{1}{4}$.
   Next, we pick a ball at random; the chance that the ball is black is (using conditional probabilities) $1 \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{4} = \frac{2}{3}$. However, if there is probability $\frac{2}{3}$ that a ball, chosen randomly from three, is black, then there must be two black and one white, which is to say that originally three was one black and one white ball in the urn.
6) *The prisoner's dilemma*. The release of two out of three prisoners has been announced. but their identity is kept secret. One of the prisoners considers asking a friendly guard to tell him who is the prisoner other than himself that will be released, but hesitates based on the following rationale: at the prisoner's present state of knowledge, the probability of being released is $\frac{2}{3}$, but after he knows the answer, the probability of being released will become $\frac{1}{2}$, since there will be two prisoners (including himself) whose fate is unknown and exactly one of the two will be released. What is wrong with this line of reasoning?
7) Problems 28 and 29 from Chapter 1 of the textbook. **Note that these questions are for practice, review, or optional study purposes and do not need to be submitted.**