Probability: Homework Set Four, March 20, 2026
Due: April 7, 2026

1) For what values of the constant C do the following define probability mass functions (PMFs) on the positive integers $1,\ 2,\ \cdots$?
   a) Geometric: $f(x) = C2^{-x}$.
   b) Logarithmic: $f(x) = C2^{-x}/x$.
   c) Inverse square: $f(x) = Cx^{-2}$.
   d) 'Modified' Poisson: $f(x) = C2^{x}/x!$.
2) For a random variable X having (in turn) each of the four probability mass functions of Problem 1), find
   a) $\mathbf{P}(X > 1)$,
   b) the most probable value of X,
   c) the probability that X is even.
3) Is it generally true that $\mathbf{E}[1/X] = 1/\mathbf{E}[X]$? Is it ever true that $\mathbf{E}[1/X] = 1/\mathbf{E}[X]$?
4) If X takes non-negative integer values and $\mathbf{E}[X]$ exists show that

$$\mathbf{E}[X] = \sum_{n=0}^{\infty} \mathbf{P}(X > n).$$

5) A random variable X has a *Pascal* (or *negative binomial*) distribution if

$$\mathbf{P}(\mathbf{X} = k) = \binom{-n}{k} p^n (-1+p)^k = \binom{n+k-1}{k} p^n (1-p)^k,\ k = 0, 1, \cdots$$

   Show that
   a) $\mathbf{E}[X] = \dfrac{n(1-p)}{p}$,
   b) $\sigma_X^2 = \dfrac{n(1-p)}{p^2}$.