Probability: Homework Set Six, April 11, 2026
Due: April 24, 2026

1) Let X and Y be independent random variables with common cumulative distribution functions F and probability density function f. Show that $V = \max\{X, Y\}$ has cumulative distribution function $\mathbf{P}(V \leq x) = F(x)^2$ and probability density function $f_V(x) = 2f(x)F(x)$, $x \in \mathbb{R}$. Find the probability density function of $U = \min\{X, Y\}$
2) Let X and Y be independent continuous random variables. Show that

$$\mathbf{E}[g(X)h(Y)] = \mathbf{E}[g(X)]\mathbf{E}[h(Y)], \tag{1}$$

whenever these expectations exist. If X and Y have the exponential distribution with parameter 1, find $\mathbf{E}[e^{(X+Y)/2}]$.
3) Suppose that x_u is the u percentile of the random variable X, that is, $F(x_u) = u$. Show that if the probability density function $f(-x) = f(x)$, then $x_{1-u} = -x_u$
4) We measure for resistance R of each resistor in a production line and we accept only the units the resistance of which is between 96 and 104 ohms. Find the percentage of the accepted units
   a) if R is uniform between 95 and 105 ohms;
   b) if R is normal with $\mu = 100$ and $\sigma = 2$ ohms. You need to use the **standard normal table**.
5) The random variable X is $N(10, 1)$. $A = \{x|(x-10)^2 < 4, x \in \mathbb{R}\}$ which is an event. Find $f_{X|X\in A}(x)$.
6) Problems 3, 4, and 14 from Chapter 3 of the textbook. **Note that these questions are for practice, review, or optional study purposes and do not need to be submitted.**