Probability: Homework Set Seven, May 5, 2026
Due: May 19, 2026

1) Let X and Y be independent discrete random variables, X being equally likely to take any value in $\{0, 1, \cdots, m\}$, and Y similarly in $\{0, 1, \cdots, n\}$. Find the probability mas function of $Z = X + Y$.
2) Let X_1 and X_2 be independent exponential random variables, parameter λ, and let $X = X_1$ and $S = X_1 + X_2$. We also know that X and S have the joint probability density function $f_{X,S}$ as

$$f_{X,S}(x, s) = \lambda^2 e^{-\lambda s} \quad \text{if } 0 \leq x \leq s. \tag{1}$$

   Suppose we know that $S = s$. What are the conditional cumulative distribution function and the conditional probability density function of X, given $S = s$?
3) Let X be uniformly random variable on $[0, \frac{1}{2}\pi]$. Find the probability density function of $Y = \sin(X)$.
4) Let X and Y be independent random variables having the exponential distribution with parameter λ and μ, respectively. Find the probability density function of $X + Y$.